Special Session on Integrating Constraint Programming and Operations Research
ISAIM 2016

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Abstracts

Grammar-Based Integer Programming Models and Methods for Employee Scheduling Problems

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We present an overview of several models and methods for solving employee scheduling problems. All approaches make use of context-free grammars to represent the rules for the composition of work shifts. We focus on the multi-activity shift scheduling (the allocation of shifts and breaks to several employees on a given day) and tour scheduling (the allocation of work days to several employees over a given planning horizon) problems. Both the anonymous (employees are identical) and the personalized (employees have different preferences and skills) cases are considered. Decomposition methods based on column generation and Benders approaches are presented.

Detecting and Exploiting Global Structures in MIP

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Current state-of-the-art MIP technology lacks a powerful modeling language based on global constraints, a tool which has long been standard in constraint programming. In general, even basic semantic information about variables and constraints is hidden to the underlying solver. For this reason, it has become standard practice in MIP implementations to devise algorithms that

basically try to reverse-engineer combinatorial substructures from a flat list of linear inequalities. In the present talk we will overview several classes of global structures that are of interest to MIP solvers, how they can be reconstructed from a flat model, and how their knowledge can be exploited to improve the performance of the solver. We will also comment on the limitations of the current approach.

Model Combinators in the Scheduling Domain

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Recent work in model combinators, as well projects like G12 and SIMPL, have achieved significant progress in automating the generation of complex and hybrid solvers from high-level model specifications. This talk expands previous work on model combinators into the scheduling domain. This is of particular interest as recent work has shown that both Constraint Programming (CP) and Mixed-Integer Programming (MIP) perform well on scheduling problems providing different capabilities and trade-offs. The ability to construct hybrid scheduling solvers to leverage the strength of both technologies as well as multiple problem encodings through high-level model combinators provides new opportunities for scheduling solvers.

Solving Non-Linear Pseudo-Boolean Optimization Problems by Constraint Integer Programming

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Pseudo-Boolean Optimization is a generalization of Binary Programming that also allows terms involving products of binary variables. These nonlinear constraints can be handled by methods from Satisfiability Solving, Constraint Programming, and Integer Programming: one can either solve them by linearization, by propagation, or combine both approaches. We describe a Constraint Integer Programming (CIP) approach that can be advantageous compared to a standard Mixed Integer Programming formulation. Furthermore, we introduce specialized presolving techniques for non-linear constraints that can be used to shrink the problem or even to transform non-linear to linear problems. These techniques have been implemented within the CIP framework SCIP, which is used for computational results.

Hybrid Approaches based on Lagrangian Relaxation

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Lagrangian relaxation is a fundamental technique in discrete optimization and operations research, with numerous practical and theoretical applications on a variety of problem classes. In this talk we present a novel way on how Lagrangian-based concepts can be applied to naturally link distinct solution methods, in particular mathematical programming, constraint programming, and decision diagram-based optimization. Our technique assumes a constraint programming framework where constraints are processed one at a time and can explicitly represent richer

substructures, such as an assignment subproblem. The underlying idea of the method is to perceive the constraint programming model as a natural decomposition of the problem, where each constraint reasons on its own set of variables. The communication between constraints is then achieved by introducing Lagrangian penalty costs between pairs of constraints. The role of these penalties is to force linking conditions among constraints, such as that the variable assignments in each of the constraints should correspond to one another. The multipliers are then used to deduce new constraints (or cuts) by performing sensitivity analysis on a linear programming or a generic decision diagram relaxation of the problem.
We discuss theoretical properties of this model, and show that propagating Lagrangian cost information can help improve the overall bound computation as well as the solution time on scheduling and other classes of problems.

Decompositions based on Decision Diagrams

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Decision diagrams have recently been investigated as a data structure for representing solutions to optimization problems and, ultimately, as the main mechanism by which general discrete optimization problems are solved. The basis for these algorithms is the use of limited-width relaxed decision diagrams, which supply an approximation of the set of feasible solutions and provide relaxation bounds, much like a linear programming relaxation in mathematical programming. Although the technique has been successful on a broad range of problems, there are problem classes in which the methodology is ineffective when compared to other generic exact approaches, as the corresponding relaxed decision diagram would have to be unreasonably large for obtaining comparatively good bounds. To overcome this shortcoming, this talk describes a new decomposition approach where small-sized decision diagrams exactly represent different portions of an optimization problem, all of which are linked through channeling constraints. Several decomposition examples will be presented, with each case elucidating a different type of decomposition that can be performed based on this concept. Finally, a few potential techniques for reasoning and solving over multiple decision diagrams will be introduced.

An Integrated Solver for Multi-index Assignment

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We study the axial and planar multi-index assignment problems under the lens of the (k,s)-assignment problem, i.e., a framework encompassing all assignment structures.
To accommodate the complementary strengths of optimization methods, we develop for the problem at hand an integrated solver that offers constraint propagation, problem-specific cuts, SoS-I branching along with heuristics and meta-heuristics. Regarding constraint propagation, we introduce a mechanism that works for both axial and planar assignment problems. Problem-specific cuts include clique and odd-hole cuts, the separation algorithms of which are taken from the literature. To tackle the diversity of the structure of these problems, we employ the

Feasibility Pump heuristic and introduce a variant that employs cutting planes within each pumping cycle. To further improve a given feasible point, we present a tabu-search scheme that is applicable to all assignment problems. All these components, when integrated in a Branch & Cut algorithm for the 3-index axial and planar cases, reduce significantly both the time and the number of nodes in the search tree compared to a competitive commercial solver. This effect is more evident for larger instances and for planar assignment in which the number of constraints increases quadratically compared to the axial one. Apart from known literature instances, we generate and examine large-size instances for which the commercial solver runs out of memory before reducing substantially the integrality gap; that is, our solver allows for much larger instances to be tackled. Further research includes testing this algorithm on 4-index assignment problems or other assignment variants.

Decomposition Methods for the Travelling Purchaser Problem

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We present novel branch-and-check and logic-based Benders decomposition techniques for the Travelling Purchaser Problem, an important optimization problem with applications in vehicle routing, logistics and warehouse management. The approach is formulated as a hybrid decomposition model with a mixed-integer linear programming master problem and two specialized cut-generating subproblems. The master problem produces a set of markets that satisfy product purchase requirements. The first subproblem generates cuts based on solution travel cost using a specialized Travelling Salesman Problem solver. The second subproblem generates subtour elimination cuts using a subtour detection algorithm and variable generation. We implement these decomposition methods on the uncapacitated asymmetric and symmetric variants of the problem and compare the performance to state-of-the-art techniques, including a constraint programming approach. Our results show that the proposed branch-and-check technique outperforms all previous approaches on the uncapacitated asymmetric variant, finding optimal solutions to previously unsolved instances. This technique also remains competitive on the uncapacitated symmetric variant for smaller problems within the benchmark set. We investigate the impact of instance symmetricity on our algorithm, and propose future extensions to the research.

Logic-Based Benders Decomposition for Multiagent Scheduling with Sequence-dependent Costs

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Logic-based Benders decomposition (LBBD) has a natural application to multi-agent scheduling problems in which the master problem assigns agents to jobs, and the subproblem schedules the jobs. The success of LBBD depends, however, on the identification of strong Benders cuts and subproblem relaxations for inclusion in the master problem. Sequence- dependent costs in the subproblem pose a particular challenge to both. Such costs occur, for example, when there are sequence-dependent setup times between jobs, or when agents are routed from one customer to another. We investigate strategies for cut generation and relaxation in this context and apply them to the routing and scheduling of home hospice care personnel. The master problem is solved by mixed integer programming and the subproblem by constraint programming. We find that with properly chosen cuts and relaxations, LBBD is superior to pure mixed integer programming and can scale up to instances of realistic size.